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# Stability of Trapped Electron Modes in Tokamaks with Elongated Cross Section

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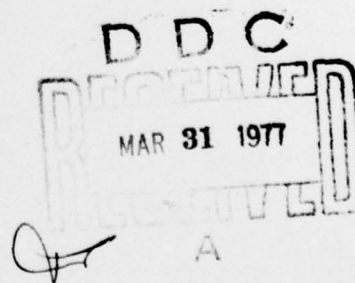
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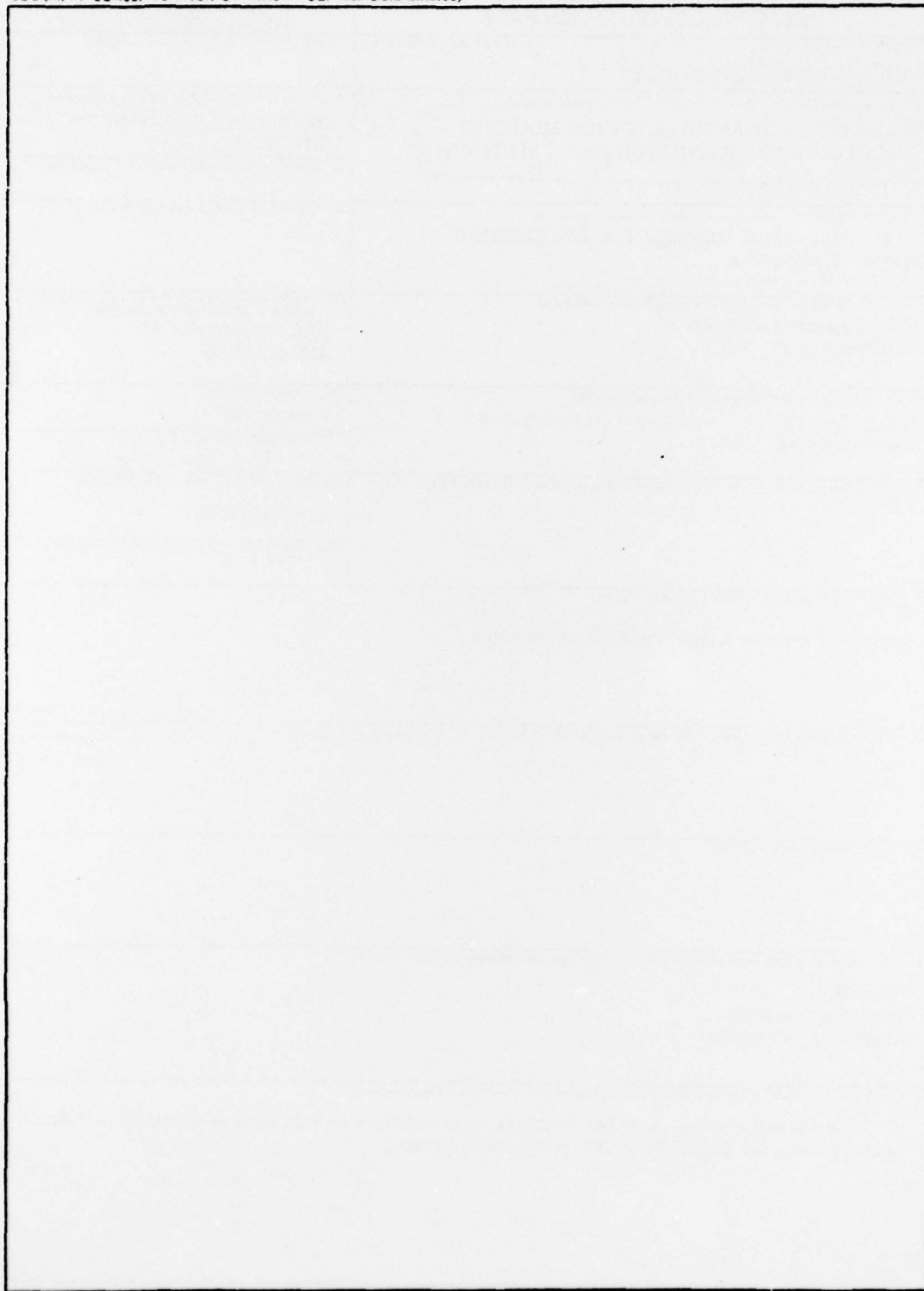
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## STABILITY OF TRAPPED ELECTRON MODES IN TOKAMAKS WITH ELONGATED CROSS SECTION

Adam, Tang, and Rutherford<sup>1</sup> have recently demonstrated that the toroidal drift of trapped electrons can lead to severe destabilization of the dissipative trapped electron mode, especially in high temperature regimes where the trapped electron collisional scattering is small. This destabilization may be viewed as resulting from a wave-particle resonance occurring when the toroidal component of the wave phase velocity is equal to the trapped particle toroidal drift. On the other hand, Glasser, et al.,<sup>2</sup> have shown that the cross-sectional elongation of a tokamak can slow down or reverse the toroidal trapped particle drift. Thus we expect a stabilizing effect on the drift resonances. In addition, non-circularity might also be expected to enhance ion Landau damping stabilization (a similar result applies to the trapped ion mode<sup>3</sup>).

The dispersion relation in noncircular cross-section axisymmetric devices will be formulated and studied in a more complete future study, Ref. 4. Here we report on some preliminary illustrative results. The full dispersion relation is the determinant of an infinite matrix each element of which is an infinite sum involving plasma dispersion functions which result from the ion orbits. Here we adopt the approximation  $(\omega R q)^2 \gg v_i^2$  valid for  $T_e \gg T_i$  ( $R$  is the major radius,  $q$  is the safety factor and  $v_i$  is the ion thermal speed) so that the plasma dispersion function arguments may be assumed to be large (thus ion Landau damping effects are not included). Also, we neglect poloidal mode structure effects and truncate the infinite matrix to a one by one (a similar approximation is commonly used for circular cross

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section treatments, e.g., Ref. 1). With these approximations the local dispersion relation becomes

$$1 + \tau - \left(\tau + \frac{\omega_*}{\omega}\right) \langle S(b) \rangle - \left\langle \int_T d^3v \frac{\omega - \omega_{*e}}{\omega + i\nu_{\text{eff}} - nV_d/R} \right\rangle = 0, \quad (1)$$

where  $S(b) = \exp(-b^2/2)I_0(b^2/2)$ ,  $b = \rho_i n B_T / R B_p(\theta)$ ,  $\rho_i$  is the ion Larmor radius,  $B_T$  and  $B_p$  are the toroidal and poloidal components of magnetic field,  $n$  is the toroidal mode number,  $\tau = T_e/T_i$ ,  $\omega_* = -ncT_e e^{-1} d(\ln N_0)/d\psi$ ,  $\omega_{*e} = \omega_* \left[1 + 0.5 \eta_e (v^2/v_e^2 - 3)\right]$ ,  $\psi$  is the magnetic flux function  $d\psi = R B_p d\lambda_\psi$ ,  $\eta_e = [d(\ln T_e)/d\psi]/[d(\ln N_0)/d\psi]$ ,  $\langle F \rangle \equiv \oint F d\theta / \oint d\theta$  where  $\theta$  is a variable denoting the position on a magnetic surface in the cross sectional plane  $d\theta = B_T [q R B_p]^{-1} d\lambda_\theta$ ,  $\int_T$  denotes integration over the trapped portion of electron velocity space,  $\nu_{\text{eff}}(v)$  is the effective collisional detrapping frequency for trapped electrons, and  $V_d(v, \lambda)$  is the toroidal drift velocity of trapped electrons which depends on both the electron velocity ( $v$ ) and pitch angle as specified by  $\lambda = \mu (\frac{1}{2} m_e v^2)^{-1}$  with  $\mu = \frac{1}{2} m_e v_\perp^2 B^{-1}$ .

We have evaluated Eq. (1) for the particular analytical model equilibrium used by Glasser, et al.<sup>2</sup>; namely, an equilibrium in which the magnetic surfaces are nested ellipses of the same ellipticity,  $\kappa$ , and the toroidal current density is constant. Figure 1 shows the function  $h(\lambda) = V_d(v, \lambda)/v^2$  as a function of the particle pitch angle variable,  $\lambda$ , for several different values of ellipticity,  $\kappa$ , on a magnetic surface of minor cross-sectional radius  $\rho = 0.25R$ . Note that for  $\kappa \geq 3.5$  all trapped particles have negative toroidal drift.

Thus for  $\kappa > 3.5$  it is not possible to satisfy the drift resonance  $\omega_r = n V_d/R$  since  $\omega_r/n$  (which we find is always positive) and  $V_d/R$  have opposite signs. Figure 2(a) shows plots of the growth rate maximized over mode number versus electron temperature for several different values of  $\kappa$  with  $T_e/T_i = 3$  (cf. caption for other parameters). Figure 2(b) shows  $\gamma$  and  $\omega_r$  versus  $\langle S(b) \rangle$  and  $\bar{k}_1 \rho_i$ , where  $\bar{k}_1 \rho_i$  is defined as  $s(\bar{k}_1 \rho_i) = \langle S(b) \rangle$ . From Figure 2(a) we note that for  $\kappa=2$  and  $\kappa=3$  the maximum growth rates are reduced by factors of about 0.7 and 0.5, respectively, as compared to the circular case ( $\kappa=1$ ), and this is approximately independent of electron temperature. Although these growth reductions are modest they may still be significant since they imply that the amount of shear necessary to stabilize the mode is correspondingly reduced.<sup>5</sup> In contrast for  $\kappa=4$  all trapped particles have their toroidal drifts reversed, and here the behavior of the maximum growth rate with temperature is qualitatively different. In particular, for  $\kappa=4$ , as the temperature is increased, the maximum growth drops dramatically since no particles are available for drift resonance and since the collisional trapped electron scattering decreases. For example, the maximum growth rate is more than two orders of magnitude less than in the circular case for  $\kappa=4$  and  $T_e = 7$  keV (actually shear would cause it to be negative).

It is interesting to note that some equilibrium studies<sup>6</sup> show that even if the elongation of the plasma boundary is modest, the interior magnetic surfaces can become very elongated.

In conclusion, if the trapped electron mode poses a significant problem for the operation of high temperature tokamaks, vertical elongation of the tokamak cross section may lead to a large increase in energy confinement times.

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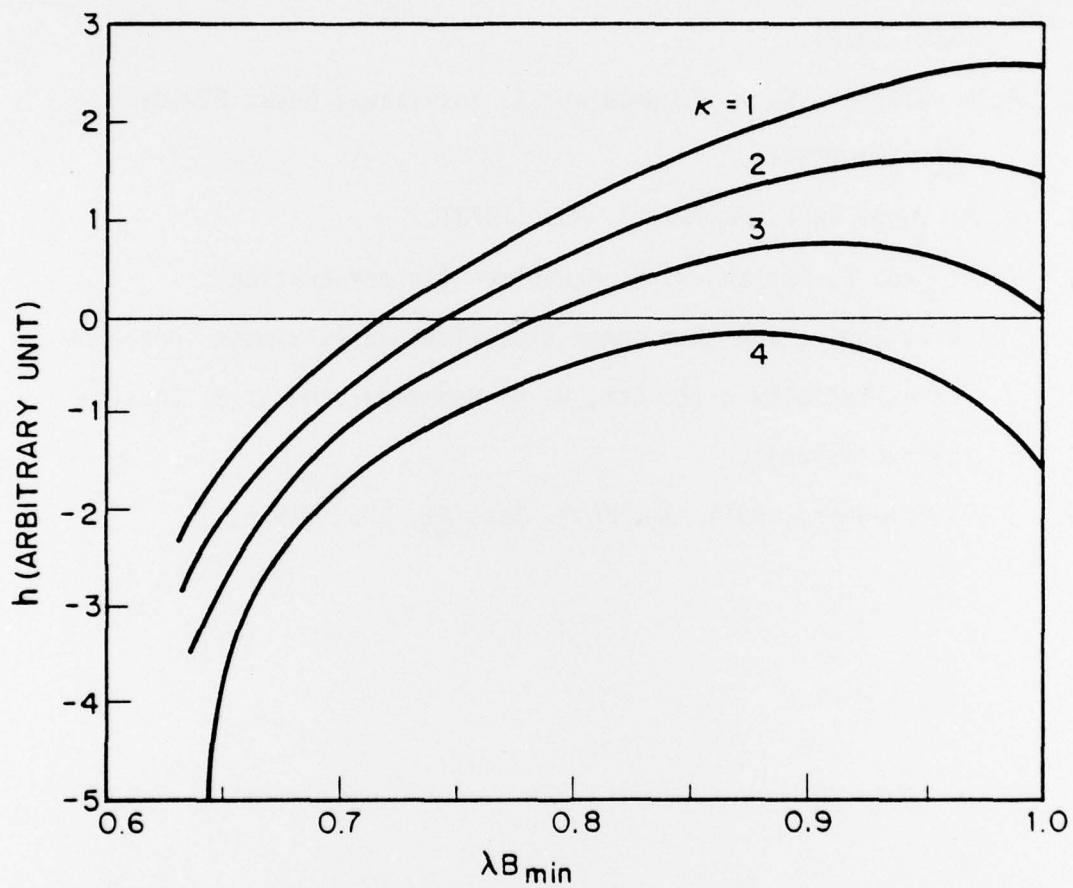


Fig. 1 —  $h(\lambda) = V_d/v^2$  (arbitrary units) versus  $\lambda B_{\min}$  for  $\rho/R = 0.25$  and  $q = 2$

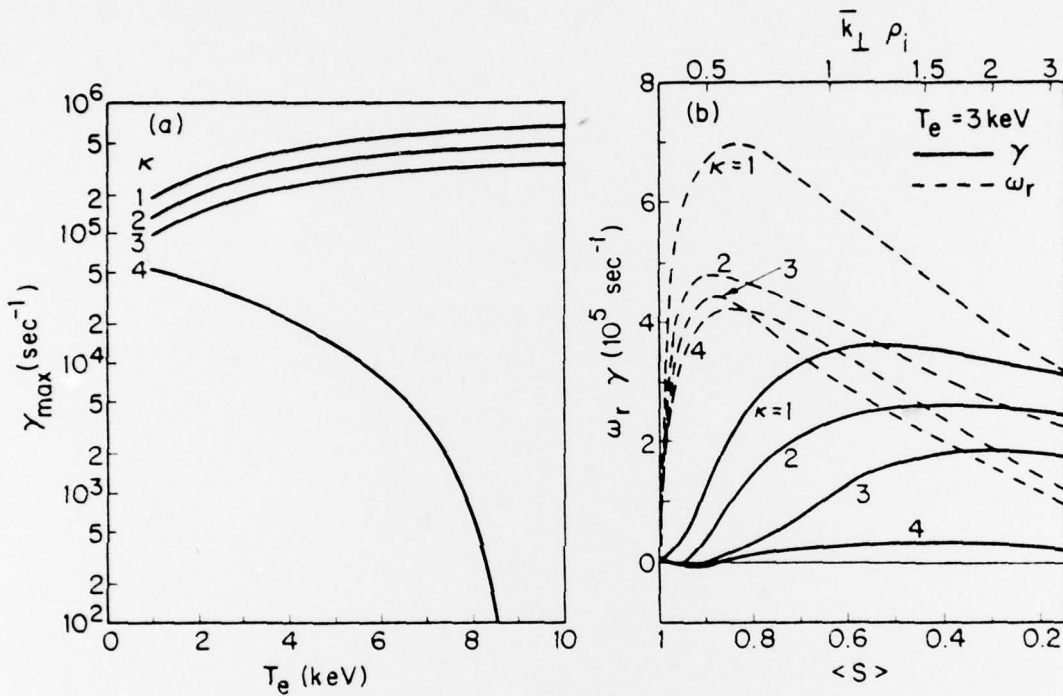


Fig. 2 — (a)  $\gamma_{\max}$  versus  $T_e$  at the wave number which gives the maximum growth rate. (b)  $\gamma$  and  $\omega_r$  versus  $\langle S(b) \rangle$  and  $\bar{k}_\perp \rho_i$  defined as  $S(\bar{k}_\perp \rho_i) \equiv \langle S(b) \rangle$ . Parameters for these figures are  $T_e/T_i = 3$ ,  $B_T = 45 \text{ kG}$ ,  $R = 130 \text{ cm}$ ,  $\rho/R = 0.25$ ,  $q = 2$ ,  $n = 5 \times 10^{13} \text{ cm}^{-3}$ ,  $Z_{\text{eff}} = 2$ ,  $\eta_e = 1$ ,  $\eta_i = 0$ ,  $L_n = -N(dN_o/d\rho)^{-1} = 20 \text{ cm}$ .

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